Problem 1: The production, P, for a given manufacturing company, measured in thousands of units, is given by the function

$$P = f(x, y) = 2x^2y^3,$$

where x is the budget for materials and y is the budget for labor (both measured in thousands of dollars). Supposing that the company has a total budget of \$2,000, find the maximum production.

Problem 2: Find the extrema of $f(x, y) = x + y$ subject to the constraint $x^2 + y^2 = 1$.

Problem 3: Find the extrema of $f(x, y) = x^2 + 2y^2$ subject to the constraint $x^2 + y^2 = 1$.

Problem 4: Find the minimum value of

$$f(x, y, z) = 2x^2 + y^2 + 3z^2$$

subject to the constraint $2x - 3y - 4z = 49$.